Smart Sensors for IOT – Exam Problem

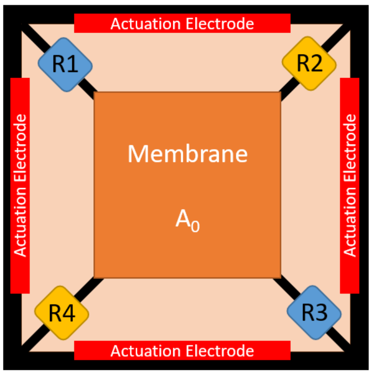
The drawing below represents a possible combined MEMS pressure sensor and gas sensor on silicon membrane.

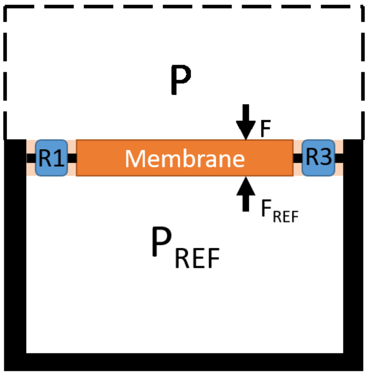
The silicon membrane resonates at the resonant frequency f_0, which is defined as:

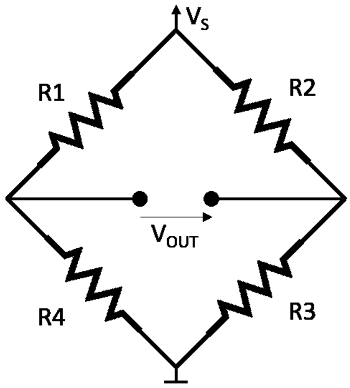
$$f_0 = \sqrt{\frac{E}{4\rho A_0}}$$

where E denotes the Young's modulus of silicon (180 GPa), ρ is the density (2330 kg/m^3) and A_0 is the area of the membrane ($100\ \mu m^2$). The thickness of the membrane is $t = 1\mu m$. A very thin polymer, sensitive to a specific gas, is deposited on top of the membrane. The polymer mass is only 5% of the mass of the silicon membrane and is not changing the equivalent stiffness of the MEMS membrane. The mass loading by the absorbed gas in the polymer on the resonant structure causes a downshift of the resonant frequency of the membrane.

The MEMS pressure sensor is based on strain-gauge elements arranged in a Wheatstone Bridge topology. The rigid membrane is suspended to the four walls of a hollow cavity by four strain gauges oriented such that for any vertical deflection of the membrane (due to a pressure differential $\Delta P = P - P_{REF}$) adjacent gauges will have opposite but equal resistance changes, and opposing gauges will have identical resistance changes. As such, when the membrane is pressed, R_1 and R_3 will **decrease** by the same amount, while R_2 and R_4 will **increase** by the same amount. This amount will be notated ΔR. Under no differential pressure stimulus, in other words when $P = P_{REF}$, all strain gauges are of equal resistance $R_{1,2,3,4} = R_0 = 100k\Omega$







The membrane and the strain gauges are built in such a way that the cavity is air-tight from the environment, and P_{REF} is pre-sealed in the factory and assumed a constant throughout the exercise.

1) Gas sensor application
   a) Calculate the equivalent stiffness, k, of the suspended MEMS membrane
   b) Calculate the sensitivity to gas mass loading defined as the derivative of the frequency with the respect to the mass, S= $\delta f / \delta m$. By analyzing the resulting equation of the sensitivity propose a solution to improve this sensitivity by changing some of the parameters of the MEMS transducer.

2) Pressure sensor application
   a) Find the relationship between V_{OUT} and $\underline{\boldsymbol{\Delta R}}$.
   b) Calculate the sensitivity of V_{OUT} with respect to $\underline{\boldsymbol{\Delta P}}$, knowing that:

- The gauge factor of all the strain gauges is $G = 5$.
- The relationship between **net** force on the membrane and the perceived strain ε is linear and is given by $= k \cdot F_{NET}$, where $k = 10^3\ N^{-1}$.
- Supply voltage of the bridge is $V_S = 10\ V$.

Express the sensitivity both in $V/_{Pa}$ and $V/_{bar}$, remembering that $1\ bar = 10^5\ Pa$.

c) If we feed the output voltage to an ADC with full scale voltage $V_{FS} = 2\ V$, calculate the number of bits needed to achieve a measurement resolution of $\Delta P = 1\ mbar$.

Solution:

1) Gas sensor application

a) The resonance frequency of the suspended MEMS membrane is

$$f_0 = \sqrt{\frac{E}{4\rho A}} = \sqrt{\frac{180\ GPa}{4*2330\ kg\ m^{-3}*100\mu m^2}} \cong 439\ MHz$$

From $f_0 = \frac{1}{2p}\sqrt{\frac{k}{m}}$ we can now calculate the stiffness, k

$k = m * (2\pi f_0)^2 \cong 1.77 * 10^6$ N/m (neglecting the mass of the polymer).

b) The sensitivity to gas mass loading is

$$\frac{\delta f}{\delta m} = -\frac{fo}{2m} \cong -0.942\frac{MHz}{pg}$$

A way to increase the sensitivity is to have MEMS structures with higher resonance frequency at same mass, so by increasing the stiffness of the resonator material or at same stiffness to use suspended masses that are much lower.

2) Pressure sensor application

a) Bridge output is given by

$$V_{OUT} = V_S\left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right)$$

We know that R1 and R3 decrease under pressure, while R2 and R4 increase from the nominal value R_0, so we can rewrite the output voltage as a function of ΔR and R_0, as follows:

$$V_{OUT} = V_S\left(\frac{R_0 + \Delta R}{R_0 - \Delta R + R_0 + \Delta R} - \frac{R_0 - \Delta R}{R_0 + \Delta R + R_0 - \Delta R}\right)$$

The ΔR elements in both denominators cancel out, so we are left with

$$V_{OUT} = V_S\left(\frac{R_0 + \Delta R}{2R_0} - \frac{R_0 - \Delta R}{2R_0}\right)$$

We can factor out ΔR inside the big bracket and manipulate the result, and we are left with

$$V_{OUT} = V_S\frac{\Delta R}{R_0}$$

b) To find the sensitivity of V_{OUT} to ΔP, we first need the relationship between ΔR and ΔP.

We know from the course (Signal Conditioners, 27Nov2019) that strain gauge resistance can be written as $R \cong R_0(1 + G\varepsilon)$. Expanded, we can write $R = R_0 + R_0 G\varepsilon$, so in fact $\Delta R = R_0 G\varepsilon$.

We then extract ε as $= F_{NET} * k$, and $F_{NET} = \Delta P * A_0$.

At this point we can re-write $\Delta R = R_0 \cdot G \cdot k \cdot A_0 \cdot \Delta P$.

As a consequence,

$$V_{OUT} = V_S \frac{R_0 \cdot G \cdot k \cdot A_0 \cdot \Delta P}{R_0}$$

And R_0 cancels out, so $V_{OUT} = V_S \cdot G \cdot k \cdot A_0 \cdot \Delta P$.

Sensitivity of output voltage to pressure differential will be

$$S = {\partial V_{OUT}}/{\partial \Delta P} = V_S \cdot G \cdot k \cdot A_0$$

Plugging in the numbers, we get $S = 50 \cdot 10^{-7}\ {V}/{Pa}$, or $S = 0.5\ {V}/{bar}$

c) Since for each bar of ΔP we get 0.5V of differential voltage, a full-scale range of 2V would ideally allow us to measure a pressure differential of 4 bars.
The minimum resolution we aim to achieve is $\Delta P_{LSB} = 1\ mbar$.
Corresponding to this, we can find the LSB voltage as $V_{LSB} = S * \Delta P_{LSB} = 0.5 \cdot 10^{-3}\ V$.
The number of bits needed to measure V_{LSB} with a full scale of V_{FS} can be approximated by

$$N = \log_2 \left(\frac{V_{FS}}{V_{LSB}}\right) = \log_2 4000 = 11.96$$

So we need a minimum of $N = 12\ b$.